SUMMER PACKET
for Algebra students entering Geometry

Please complete every problem and SHOW ALL WORK. <u>NO WORK = NO CREDIT</u>. Write your answers on the answer sheet at the end of the packet. This assignment will be graded for both accuracy and completion. You will be tested during the second week of school on each of the following concepts.

<u>DETERMINING WHETHER A POINT IS ON A LINE</u>
<u>Example 1</u>

Decide whether (3,-2) is a solution of the equation $y = 2x - 8$

$-2 = 2(3) - 8$ *Substitute 3 for x and -2 for y.*

$-2 = -2$ *Simplify.*

The statement is true, so (3,2) is a solution of the equation $y = 2x - 8$

<u>Exercises:</u> Decide whether the given ordered pair is a solution of the equation.

1. $y = 6x + 4; (-2, 8)$ _____

2. $y = -10x - 2; (1, -12)$ _____

3. $y = -\frac{1}{4}x - 18; (-4, -17)$ _____

4. $y = \frac{3}{2}x + 10; (4,12)$ _____

5. $y = \frac{5}{9}x + 34; (-9,27)$ _____

6. $y = \frac{2}{3}x - 6; (9,0)$ _____

<u>CALCULATING SLOPE</u>
<u>Example 2</u>

Find the slope of a line passing through (3,-9) and (2,-1).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Formula for slope*

$m = \frac{-1-(-9)}{2-3} = \frac{-1+9}{-1}$ *Substitute values and simplify.*

$m = \frac{8}{-1} = -8$ *Slope is -8.*

<u>Exercises:</u> Find the slope of the line that contains the points

7. (4,1), (3, 6) _____

8. (-8, 0), (5, -2)_____

9. (5, 6), (9,8)_____

10. (0,-4), (7,3)_____

11. (-1, 7), (-3, 18)___

12. (-6, -4), (1, 10) ___

FINDING THE EQUATION OF A LINE

Example 3

Find an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

$y = mx + b$ *Write the slope-intercept form.*

$4 = 3m + 5$ *Substitute 5 for b, 3 for x, and 4 for y.*

$-1 = 3m$ *Subtract 5 from each side.*

$\frac{-1}{3} = m$ *Divide each side by 3.*

The slope is $m = \frac{-1}{3}$. The equation of the line is $y = \frac{-1}{3}x + 5$

Exercises: Write the equation of the line that passes through the given point and has the given y-intercept.

13. (2, 1); *b*=5 _____   16. (7, 0); *b*=13_____

14. (-5, 3); *b*=-12_____   17. (-3, -3); *b*=-2_____

15. (-3, 10); *b*=8_____   18. (-1, 4); *b*=-8_____

FINDING THE EQUATION OF A LINE

Example 4

Write an equation of the line that passes through the points (4, 8) and 3, 1). Find the slope of the line.

$m = \frac{1-8}{3-4}$ *Substitute values.*

$m = \frac{-7}{-1} = 7$ *Simplify.*

$1 = 7(3) + b$ *Substitute values into y = mx + b.*

$1 = 21 + b$ *Multiply.*

$-20 = b$ *Solve for b.*

Exercises: Write an equation of the line that passes through the given points.

19. (6, -3), (1, 2)_____   21. (5, -1), (4, -5)_____   23. (-3, -7), (0, 8)_____

20. (-7, 9), (-5, 3)_____   22. (-2, 4), (3, -6)_____   24. (1, 2), (-1, -4)_____

DISTANCE FORMULA

Example 5

Find the distance between
the points (-4, 3) and (-7, 8)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-7 - (-4)\right)^2 + (8 - 3)^2}$$
$$= \sqrt{(-3)^2 + (5)^2}$$
$$= \sqrt{34}$$

Exercises: Find the distance between the points

25. (3, 6), (0, -2)_____

27. (-3, 4), (1, 4)_____

29. (8, -2), (-3, -6)_____

26. (5, -2), (-6, 5)_____

28. (-6, -6), (-3, -2)_____

30. (-8, 5), (-1, 1)_____

COMBINING LIKE TERMS

Example 6

Simplify

$$8x^2 + 16xy - 3x^2 + 3xy - 3x$$
$$8x^2 - 3x^2 + 16xy + 3xy - 3x \qquad \text{Group like terms}$$
$$5x^2 - 3x + 19xy \qquad \text{Simplify}$$

Exercises: Simplify.

31. $6x + 11y - 4x + y$

33. $-3p - 4t - 5t - 2p$

35. $3x^2y - 5xy^2 + 6x^2y$

32. $-5m + 3q + 4m - q$

34. $9x - 22y + 18x - 3y$

36. $5x^2 + 2xy - 7x^2 + xy$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

Example 7

Solve. $6a - 12 = 5a + 9$

$a - 12 = 9$ *Subtract 5a from both sides*

$a = 21$ *Add 12 to each side*

Exercises: Solve the equation.

37. $3x + 5 = 2x + 11$

38. $8m + 1 = 7m - 9$

39. $11q - 6 = 3q + 8q$

40. $-14 + 3a = 10 - a$

41. $-2t + 10 = -t$

42. $-7x + 7 = 2x - 11$

SOLVING INEQUALITIES

Example 8   Solve.

a. $5x - 4 \geq 4x + 6$ b. $10 - 7x < 24$

When you multiply or divide each side of an inequality by a *negative* number, you must *reverse* the inequality symbol to maintain a true statement.

a. $5x - 4 \geq 4x + 6$
$x - 4 \geq 6$
$x \geq 10$

b. $10 - 7x < 24$
$-7x < 14$
$x > -2$

Exercises: Solve the inequality.

43. $-x + 2 > 7$

44. $-5 + m < 21$

45. $z + 6 > -2$

46. $c - 18 < 10$

47. $x - 5 < 4$

48. $-3x + 4 \leq -5$

WRITING AND SIMPLIFYING RATIOS

Example 9

a. Train A takes 35 minutes to travel its route. Train B, traveling the same route but making more stops, takes 47 minutes. What is the ratio of the time of Train A to Train B?

b. Jennie's height is 4 feet, 7 inches. Her younger sister's height is 25 inches. Find the ratio of Jennie's height to her sister's.

Solutions

a. 35 minutes to 47 minutes $= \frac{35 \text{ minutes}}{47 \text{ minutes}} = \frac{35}{47}$

b. Convert 4 feet, 7 inches to inches: $4(12) + 7 = 55$ inches

55 inches to 25 inches $= \frac{55 \text{ inches}}{25 \text{ inches}} = \frac{55}{25} = \frac{11}{5}$

Exercises: Write the following ratios.

49. Basmati rice needs to cook for 20 minutes, while quinoa (another grain) cooks for 25 minutes. What is the ratio of cooking times for rice to quinoa?

50. Jonathan caught 7 fish and Geogeanne caught 4. What is the ratio of fish caught of Jonathan to Georgeanne?

51. Two sunflowers' growth was measured daily. At the end of the experiment, Sunflower A had grown from 2 inches to 2 feet, 3 inches. Sunflower B had grown from 3 inches to 2 feet, 6 inches. Find the ratio of the growth in height of Sunflower A to Sunflower B.

Use the diagram at the right.
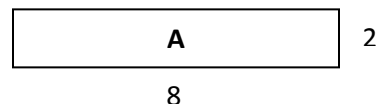
52. What is the ratio of length to width of rectangle A?

53. What is the ratio of the perimeter of rectangle A to the perimeter of rectangle B?

54. What is the ratio of the area of rectangle A to the area of rectangle B?

| A | 2 |
| 8 | |

| B | 4 |
| 4 | |
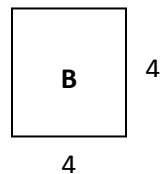
DISTRIBUTIVE PROPERTY

Example 10

Solve.

a. $4(x + 3) = 36$
$4x + 12 = 36$
$4x = 24$
$x = 6$

b. $6(x + 4) + 12 = 5(x + 3) + 7$
$6x + 24 + 12 = 5x + 15 + 7$
$6x + 36 = 5x + 22$
$x = -14$

Exercises: Solve.

55. $2(x + 7) = 20$

56. $-10(y + 8) - 40$

57. $7(2 - x) = 5x$

58. $-4(x - 6) = 28$

SOLVING PROPORTIONS

Example 11

Solve.

a. $\dfrac{x}{8} = \dfrac{3}{4}$

$4x = 8 \cdot 3$

$4x = 24$

$x = 6$

b. $\dfrac{6}{x+4} = \dfrac{1}{9}$

$6 \cdot 9 = x + 4$

$54 = x + 4$

$50 = x$

Exercises: Solve.

59. $\dfrac{y}{50} = \dfrac{3}{100}$

60. $\dfrac{6}{45} = \dfrac{2z + 10}{15}$

61. $\dfrac{3}{p - 6} = \dfrac{1}{p}$

62. $\dfrac{3}{8} = \dfrac{3}{2d}$

63. $\dfrac{1}{18} = \dfrac{5}{-4(x - 1)}$

64. $\dfrac{r}{3r + 1} = \dfrac{2}{3}$

65. $\dfrac{3w + 6}{28} = \dfrac{3}{4}$

66. $\dfrac{3}{m + 4} = \dfrac{9}{14}$

67. $\dfrac{w}{4} = \dfrac{9}{w}$

SIMPLIFYING RADICALS

Example 12

Simplify the expression $\sqrt{20}$

$$\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$$
$$= 2\sqrt{5}$$

Exercises: Simplify the expression.

68. $\sqrt{121}$

69. $\sqrt{40}$

70. $\sqrt{243}$

71. $\sqrt{52}$

72. $\sqrt{27}$

73. $\sqrt{288}$

74. $\sqrt{45}$

75. $\sqrt{80}$

76. $\sqrt{320}$

77. $\sqrt{72}$

78. $\sqrt{50}$

79. $\sqrt{225}$

SIMPLIFYING RADICAL EXPRESSIONS

Example 13

a. $5\sqrt{3} - \sqrt{3} - \sqrt{2}$
$= 4\sqrt{3} - \sqrt{2}$

b. $(2\sqrt{2})(5\sqrt{3})$
$= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$
$= 10\sqrt{6}$

c. $(5\sqrt{7})^2$
$= 5^2\sqrt{7^2}$
$= 25 \cdot 7$
$= 175$

Exercises: Simplify the radical expression.

80. $\sqrt{75} + \sqrt{3}$

81. $-\sqrt{147} - \sqrt{243}$

82. $(5\sqrt{4})(2\sqrt{4})$

83. $\sqrt{50} - \sqrt{18}$

84. $(3\sqrt{14})(\sqrt{35})$

85. $(6\sqrt{5})^2$

86. $\sqrt{64} - \sqrt{28}$

87. $(\sqrt{363})(\sqrt{300})$

88. $(4\sqrt{2})^2$

89. $\sqrt{44} + 2\sqrt{11}$

90. $(\sqrt{32})(\sqrt{2})$

91. $(8\sqrt{3})^2$

92. $\sqrt{125} - \sqrt{80}$

93. $(\sqrt{98})(\sqrt{128})$

94. $(10\sqrt{11})^2$

95. $\sqrt{242} + \sqrt{200}$

SIMPLIFYING QUOTIENTS WITH RADICALS

Example 14

Simplify the quotient $\frac{6}{\sqrt{5}}$

$$\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}}$$
$$= \frac{6\sqrt{5}}{5}$$

Exercises: Simplify the quotient.

96. $\frac{4}{\sqrt{3}}$

97. $\frac{2\sqrt{3}}{\sqrt{5}}$

98. $\frac{\sqrt{32}}{\sqrt{5}}$

99. $\frac{5}{\sqrt{7}}$

100. $\frac{\sqrt{12}}{\sqrt{24}}$

101. $\frac{\sqrt{27}}{\sqrt{45}}$

102. $\frac{2\sqrt{3}}{\sqrt{6}}$

103. $\frac{\sqrt{18}}{\sqrt{10}}$

104. $\frac{\sqrt{50}}{\sqrt{75}}$

SOLVING LITERAL EQUATIONS

Example 16

Given the formula for the surface area of a right cylinder, solve for h. $S = 2\pi r^2 + 2\pi rh$

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi r} = r + h \qquad \qquad \text{or}$$

$$\qquad \qquad \qquad \qquad \qquad \qquad S - 2\pi r^2 = 2\pi rh$$

$$\frac{S}{2\pi r} - r = h \qquad \qquad \frac{(S - 2\pi r^2)}{2\pi r} = h$$

Exercises: Solve the literal equation for the indicated variable. Assume variables are positive.

105. $V = \frac{4}{3}\pi r^3; r$

106. $V = s^3; s$

107. $V = \pi r^2 h; h$

108. $A = \frac{1}{2}bh; h$

109. $P = 2l + 2w; l$

110. $S = 6s^2; s$

111. $A = \frac{1}{2}h(b_1 + b_2); b_1$

112. $V = lwh; h$

113. $a^2 + b^2 = c^2; b$

ALGEBRAIC EXPRESSIONS

Example 17

a. Write an expression for seven less than a number

$x - 7$

b. Write an equation for three times less than six times a number is five times the same number plus 5, then solve.

$$6x - 3 = 5x + 5$$
$$x - 3 = 5$$
$$x = 8$$

Exercises: Write the expression or equation. Solve the equations.

114. Half of a number plus three times the number

115. The product of five and a number decreased by seven equals thirteen.

116. Sixteen less than twice a number is 10.

117. Twice a number increased by the product of the number and fourteen results in forty-eight.

118. Half of a number is three times the sum of the number and five.

PERCENT PROBLEMS
Example 18

    a. What number is 12% of 75?

$$x = 0.12(75)$$
$$x = 9$$

    b. 6 is what percent of 40?

$$6 = 40p$$
$$0.15 = p$$
$$p = 15\%$$

Exercises:

119.   What number is 30% of 120?

120.   11 dogs is what percent of 50 dogs?

121.   What distance is 15% of 340 miles?

122.   200 is what percent of 50?

123.   34 is what percent of 136?

124.   8 weeks is what percent of a year?

SIMPLIFYING RATIONAL EXPRESSIONS
Example 19

    Simplify.

    a. $\dfrac{8x^2+12x}{4x^2+16x} = \dfrac{4x(2x+3)}{4x(x+4)} = \dfrac{2x+3}{x+4}$

    b. $\dfrac{y^2-9}{y^2+6y+9} = \dfrac{(y+3)(y-3)}{(y+3)(y+3)} = \dfrac{y-3}{y+3}$

Exercises: Simplify.

125. $\dfrac{5x}{10x^2}$

126. $\dfrac{14d^2 - 2d}{6d^2 + 8d}$

127. $\dfrac{-5h + 1}{h + 1}$

128. $\dfrac{16a^3}{8a}$

129. $\dfrac{2y - 12}{24 - 2y}$

130. $\dfrac{t^2 - 1}{t^2 + 2t + 1}$

131. $\dfrac{(5x^2 + x)}{(5x + 1)}$

132. $\dfrac{36s^2 - 4s}{4s^2 - 12s}$

133. $\dfrac{m^2 - 4m + 4}{m^2 - 4}$

ANSWER SHEET

1_____   25. _____   49. _____   73. _____

2._____   26. _____   50. _____   74. _____

3. _____   27. _____   51. _____   75. _____

4. _____   28. _____   52. _____   76. _____

5. _____   29. _____   53. _____   77. _____

6. _____   30. _____   54. _____   78. _____

7. _____   31. _____   55. _____   79. _____

8. _____   32. _____   56. _____   80. _____

9. _____   33. _____   57. _____   81. _____

10. _____   34. _____   58. _____   82. _____

11. _____   35. _____   59. _____   83. _____

12. _____   36. _____   60. _____   84. _____

13. _____   37. _____   61. _____   85. _____

14. _____   38. _____   62. _____   86. _____

15. _____   39. _____   63. _____   87. _____

16. _____   40. _____   64. _____   88. _____

17. _____   41. _____   65. _____   89. _____

18. _____   42. _____   66. _____   90. _____

19. _____   43. _____   67. _____   91. _____

20. _____   44. _____   68. _____   92. _____

21. _____   45. _____   69. _____   93. _____

22. _____   46. _____   70. _____   94. _____

23. _____   47. _____   71. _____   95. _____

24. _____   48. _____   72. _____   96. _____

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97. _____

98. _____

99. _____

100. _____

101. _____

102. _____

103. _____

104. _____

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107. _____

108. _____

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125. _____

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129. _____

130. _____

131. _____

132. _____

133. _____